On the Road to Old Town
Improving the Acadian Variant of FVS

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Introduction

Question: What do you do when you retire and have time on your hands?

Answer: Why not recode a growth model and then refit many of the equations? – *For Fun!*

The Acadian Variant (ACD)

The Acadian Variant of FVS officially exists as R[1] code and is maintained by Aaron Weiskittel and Ben Rice (among others). All the driving equations are from published literature, principally authored by:

- Christian Kuehne
- Cen Chen
- Matthew Russell
- Aaron Weiskittel
- John Kershaw
- Mark Castle

[1] R Core Team (2023). R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria. <https://www.R-project.org/>

Rationale for the Work

While ACD performs well and is demonstrably better than FVS in independent validations, there were several things bugging me:

- Use of a plot-level basal area increment equation to limit basal area growth from tree-level diameter growth estimates;
- Poor mortality trends for over-stocked stands;
- Use of a cap on height growth to regulate unreasonably high height growth rates;
- A number of bugs and inconsistencies in the R code;
- My personal obsession for execution speed; and
- My need for Type II fun.

The Work

What I did:

- Re-coded the model in C++ (to the C++ 23 standard) and built an R package interface;
- Refit:
  - diameter growth equations;
  - individual tree mortality equations;
  - height growth equations;
  - height to crown base change equations;
- Worked with Ben Rice's FIA plot benchmarking data set to validate and test the model.

What Needs to be Done (short list):

- Small tree growth;
- High expansion factor tree record management;
- Ingrowth;
- Chasing trends in residuals;
- Fully diagnosing high basal area attainment issues.

CAUTION

What follows is decidedly "in the weeds".

Raise your hand if you want me to re-emerge and go at a higher level.

Diameter Growth Equation

ACD uses individual tree diameter growth equations developed by Kuehne, et al.[2]. The model form is:

$$\Delta dbh = e^{(\beta_0+\beta_{0r})+\beta_1 log(dbh)+(\beta_2+\beta_{2r})dbh+(\beta_3+\beta_{3r})log(cr)+(\beta_4+\beta_{4r})log(bal_{sw}+0.01)+\beta_5 bal_{hw}+\beta_8 log(CSI)}$$

where parameters subscripted by r are random effects on species, basal area in larger trees (`bal`) is computed for softwoods (*sw*) and hardwoods (*hw*), and `CSI` is climate site index[3] in meters.

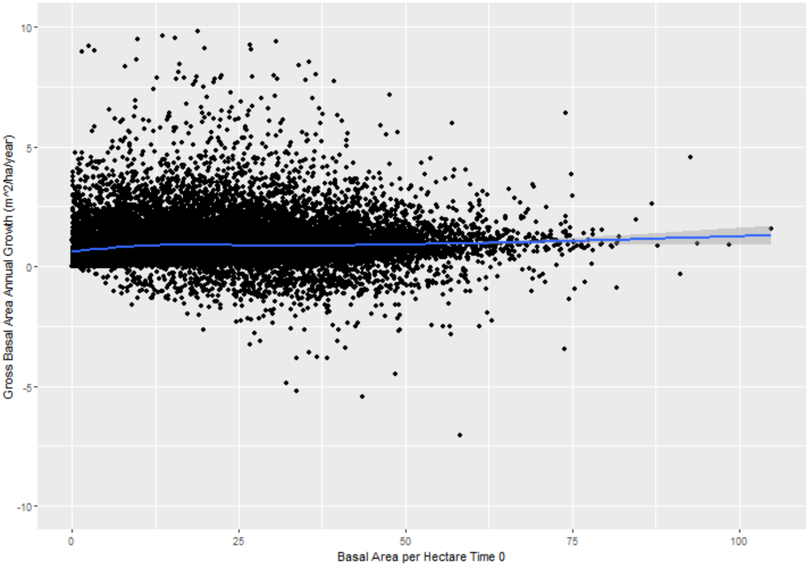
Notice that there is no basal area (`ba`) term typically used to express two-sided competition.

[2] Kuehne, C., Weiskittel, A., & Kershaw Jr., J.A. Development and Evaluation of Refined Annualized Individual Tree Diameter and Height Increment Equations for the Acadian Variant of the Forest Vegetation Simulator: Implication for Forest Carbon Estimates. Mathematical and computational forestry naturalresource sciences, 14 (2)

[3] Weiskittel, A.R.; N.L. Crookston; P.J. Radtke. 2011a. Linking climate, gross primary productivity, and site index across forests of the western United States. Can. J. For. Res. 41: 1710–1721. doi: https://doi.org/10.1139/x11-086

About That Basal Area Term

Expect the unexpected (or how does that happen?):



Revised Diameter Growth Equation

The revised diameter growth equation shares a lot with the ACD equation, however, there are several notable changes. The equation is:

$$\Delta dbh = e^{\beta_0+\beta_1 log(dbh+1.0)+\beta_2 dbh+\beta_3 log(cr)+\beta_4 bal/log(dbh+1.0)+\beta_5 log(CSI)}$$

The modifications were to add 1.0 to `dbh` in the log terms of the equation, and to replace the basal area in larger trees (`bal`) terms with a formulation used by Hann[4] involving an interaction with `dbh`.

The revised equation was fit separately to all species with ≥ 20000 observations.

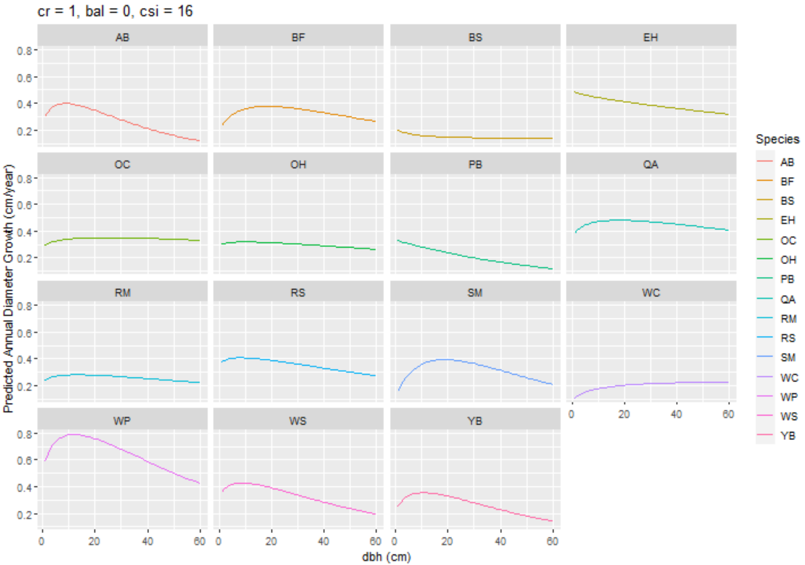
[4] Hann, David W., David D Marshall, Mark L Hanus, and Oregon State University. Forest Research Laboratory. 2006. Reanalysis of the Smc-Organon Equations for Diameter-Growth Rate, Height-Growth Rate, and Mortality Rate of Douglas-Fir. : Corvallis, OR : Forest Research Laboratory, Oregon State University.

Revised Diameter Growth Equation Parameter Estimates

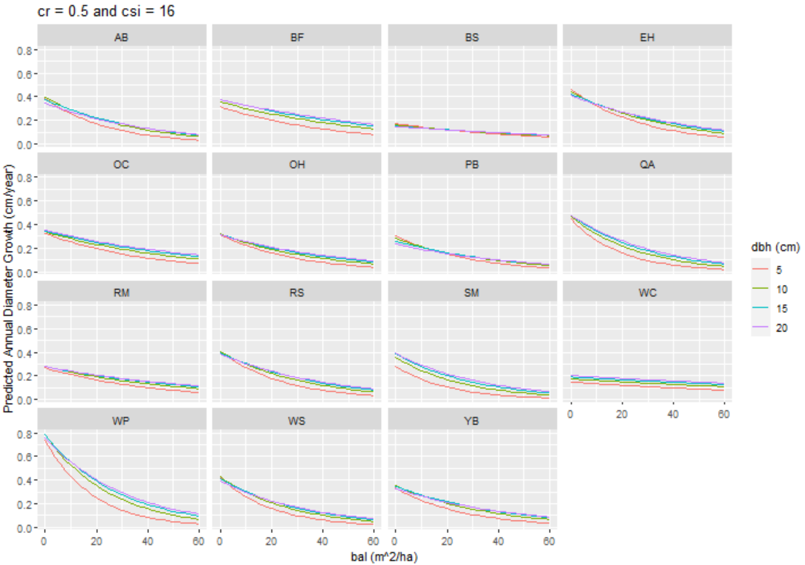
Diameter Growth Equation Parameter Estimates

FVS	N	MSE	b0	b1	b2	b3	b4	b5
BF	452673	1.24	-2.1435857	0.3593028	-0.0184791	1.0047516	-0.0410973	0.4104913
RS	244586	0.88	-1.5602407	0.1025491	-0.0114136	0.7177303	-0.0733104	0.3703701
RM	174220	1.05	-2.0742269	0.1344965	-0.0092508	0.6704055	-0.0443568	0.3731337
BS	116026	0.53	-0.8058522	-0.1504027	0.0021861	0.8818587	-0.0322017	-0.0288774
PB	81863	0.69	-1.4992348	0.0020848	-0.0171510	0.7625551	-0.0656148	0.3339626
WS	55654	1.34	-1.3865373	0.2347620	-0.0238776	0.8480158	-0.0861475	0.2881720
WC	51326	0.58	-2.1986872	0.3141367	-0.0061004	0.8876820	-0.0189920	0.1375218
SM	50383	1.06	-3.6640496	0.6651070	-0.0335220	0.5544203	-0.0898102	0.6356263
YB	42548	1.39	-1.0799670	0.3609764	-0.0305255	0.6722657	-0.0667012	-0.0188244
EH	39516	1.25	-1.1141883	-0.0193093	-0.0059281	0.7563972	-0.0634549	0.3357535
WP	32619	3.61	-0.7180888	0.2856932	-0.0221441	0.6376906	-0.0967224	0.1648024
QA	30893	1.66	-1.6835405	0.1627486	-0.0085212	0.3623651	-0.0925568	0.3160373
AB	29768	0.94	-2.6919761	0.3500209	-0.0361457	0.7539952	-0.0733272	0.6543146

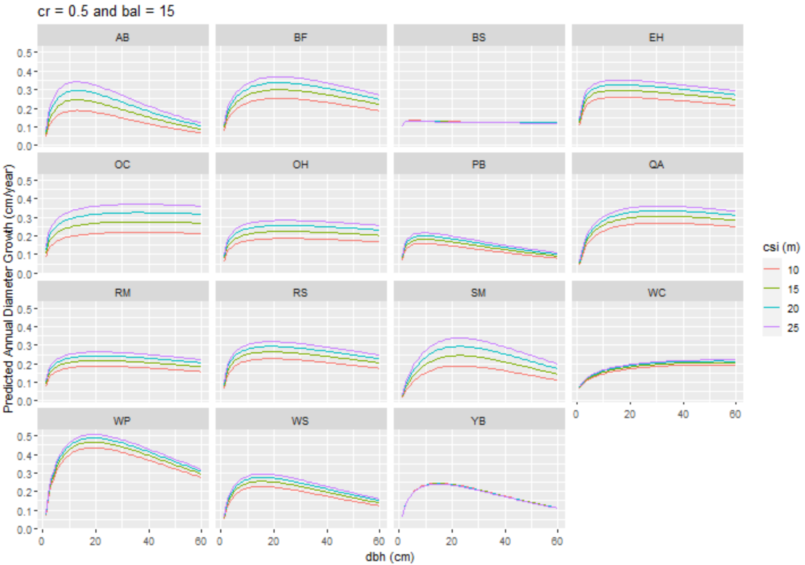
Revised Diameter Growth Equation Behavior



Revised Diameter Growth Equation Behavior



Revised Diameter Growth Equation Behavior



Height Growth Equation

ACD uses individual tree diameter growth equations developed by Kuehne, et al. The model form is:

$$\Delta ht = e^{\beta_{0r} + \beta_1 log(ht) + \beta_{2r} ht + \beta_3 cr + \beta_4 (ccfl/100) + \beta_5 csi^2}$$

where ht = total height (m), cr = crown ratio, $ccfl$ is crown competition factor in larger trees (m^2/ha), and csi is climate site index (m). Parameters subscripted by r are random effects on species.

Notice:

- The equation is not tied directly to a site curve trajectory (common in many Prognosis/Organon type models).
- Asympotic behavior is not guaranteed, but is possible depending on β_1 and β_{2r}.
- consistency with `csi` (site index) is not assured (not necessarily a bad thing).
- Site index enters as a squared term!

The equations as fit by Kuehne et al. required caps on growth estimates to produce reasonable height attainment. These caps are somewhat arbitrary.

Revised Height Growth Equation

The revised equations were designed to:

1. Enforce biologically reasonable long-term behavior (i.e., limit total height attainment to observed maxima),
2. Demonstrate peaking or monotonically decreasing height growth estimates over initial total height, and
3. Incorporate predictor variables capturing social position or competitive status.

Revised Height Growth Equation

We took the basic Chapman-Richards equation form[5] which produces an asymptote as the integrated framework for the height growth equation. The equation differentiated with respect to `ht` is:

$$\Delta ht = abc * e^{-bht}(1 - e^{-bht})^{(c-1)}$$

where `a` is the asymptote, and `b` and `c` are the slope and shape parameters. Using this framework to enforce an asymptotic height, we took data provided by Ben Rice to supply an maximum height for each species (ht_{max}). The ht_{max} becomes `a` in the differentiated equation.

[5] Pienaar, L. V. and Turnbull, K. J., 1973. The Chapman-Richards generalization of von Bertalanffy's growth model for basal area growth and yield in even-aged stands. Forest Science 19: 2-22.

Revised Height Growth Equation

The revised equation:

$$\Delta ht = ht_{max}\beta_1\beta_2 cr^{\beta_3}(csi/30)^{\beta_5} e^{(-\beta_1 ht - \beta_4(ccfl/100))}(1 - e^{-\beta_1 ht})^{(\beta_2 - 1)}$$

The `ccfl` term was inserted into the equation in the first exponential term as that term generally defines the decay of height growth past the peak where competition effects would most likely occur.

We used a cut-off of 3000 observations to limit the species fit with the equation.

Revised Height Growth Equation

Conifers were fit separately by species, whereas hardwoods were fit with a mixed model where β_4 and β_5 were random effects on species. Other conifers parameters were estimated using a combined data set with all conifer species. Other hardwoods parameters were taken from the fixed effects estimates from the mixed model.

Estimates for White Pine (`WP`) produced errant behavior. `WP` parameters were set to the other conifers estimates with the β_0 (maximum height) set to the `WP` value.

The mixed model for hardwoods resulted in a consistent under-prediction bias. We corrected this by estimating a correction ratio applied to the β_0 parameter for each species i:

$$\beta_{0a,i} = \beta_{0,i} \Sigma(\frac{\Delta ht_i}{\widehat{\Delta ht_i}})/n_i$$

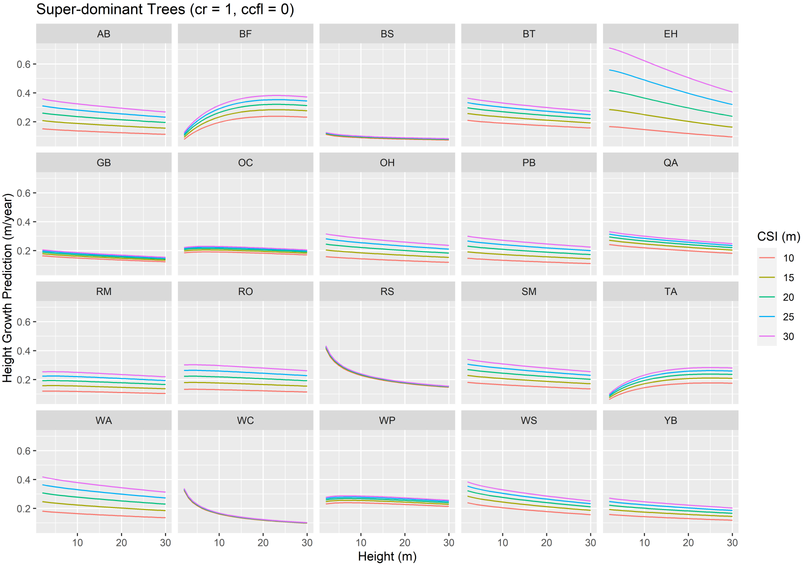
where $\beta_{0a,i}$ is the adjusted parameter for each species.

Revised Height Growth Equation

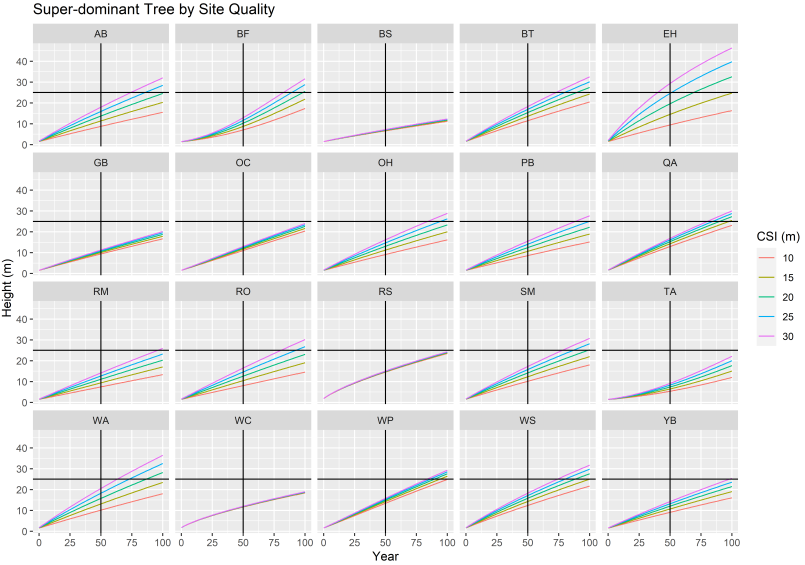
Height Growth Equation Parameter Estimates

FVS	N	MSE	b0	b1	b2	b3	b4	b5
BF	218303	1.57	30.18293	0.0237654	1.7309872	0.3396236	0.0000000	0.4300000
RS	148464	1.19	35.21341	0.0031977	0.6452656	0.2798559	0.0000000	0.0453984
BS	52318	0.76	25.60976	0.0029867	0.8749023	0.1937343	0.0000000	0.1061788
WS	44443	1.62	30.94512	0.0113415	0.9594453	0.2612782	0.0000000	0.4300000
WP	21934	2.46	39.63415	0.0086709	1.0633682	0.2774026	0.0000000	0.1660177
EH	20593	1.52	35.06098	0.0226999	1.0317121	0.2262924	0.0000000	1.3184268
WC	17167	1.30	31.40244	0.0016327	0.5722277	0.1250221	0.0000000	0.0407049
TA	11039	1.86	26.06707	0.0199065	1.6493001	0.3596588	0.0000000	0.4300000
OC	534261	0.00	31.72659	0.0086709	1.0633682	0.2774026	0.0000000	0.1660177
AB	15337	2.27	40.11061	0.0085548	0.9805647	0.0000000	0.0576520	0.7815698
BT	5494	4.98	40.83646	0.0085548	0.9805647	0.0000000	0.0076990	0.5001779
GB	3741	2.13	23.13511	0.0085548	0.9805647	0.0000000	0.0455226	0.2042321
PB	40823	2.50	33.69588	0.0085548	0.9805647	0.0000000	0.0405947	0.6465336
QA	11060	5.45	37.15198	0.0085548	0.9805647	0.0000000	0.0092467	0.2840619
SM	29398	3.17	38.14175	0.0085548	0.9805647	0.0000000	0.0405047	0.5744540
WA	5119	3.68	46.89629	0.0085548	0.9805647	0.0000000	0.0336856	0.7608173
YB	29176	2.99	30.45648	0.0085548	0.9805647	0.0000000	0.0185188	0.4918140
RM	126899	2.59	34.96240	0.0081491	1.0320316	0.0000000	0.0000000	0.6855181
RO	9278	3.42	41.56113	0.0081491	1.0320316	0.0000000	0.0000000	0.7522605
OH	276325	0.00	35.41047	0.0085548	0.9805647	0.0000000	0.0164149	0.6245655

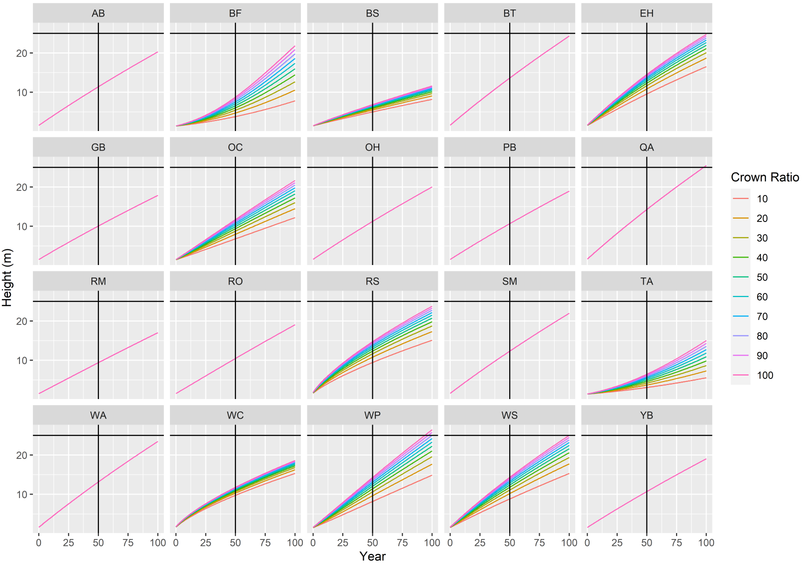
Revised Height Growth Equation Behavior



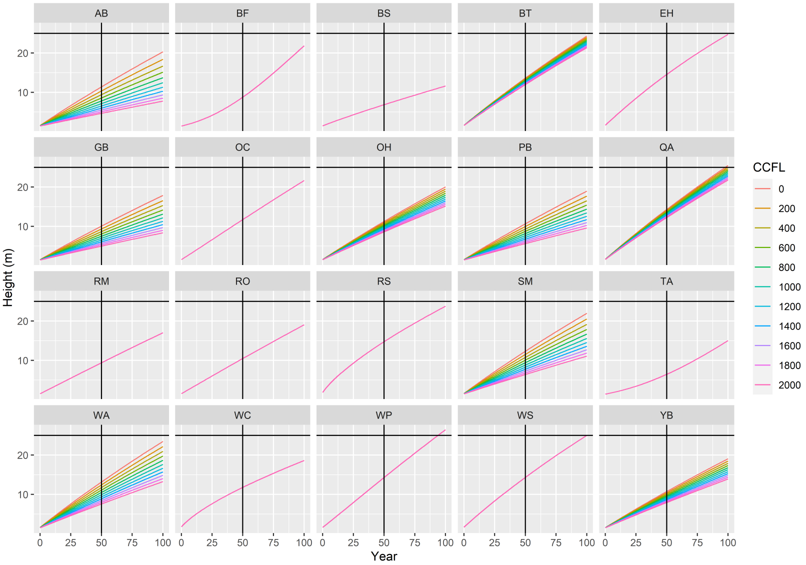
Revised Height Growth Equation Behavior



Revised Height Growth Equation Behavior



Revised Height Growth Equation Behavior



Individual Tree Survival/Mortality

ACD uses a multi-level approach to predicting individual tree mortality developed by Chen, et al.[6] This approach uses individual tree mortality estimates to allocate plot-level mortality estimates.

Experience with the system showed that mortality for highly stocked stands was under-estimated. Several workarounds were employed in ACD.

6 Cen Chen, John Kershaw Jr, Aaron Weiskittel, Elizabeth McGarrigle. 2023. Can a multistage approach improve individual tree mortality predictions across the complex mixed-species and managed forests of eastern North America?, Forest Ecosystems, Volume 10, 2023, 100086, ISSN 2197-5620, https://doi.org/10.1016/j.fecs.2023.100086.

Revised Individual Tree Survival

We fit the following survival equation (probability of survival) of a Gompit form to observations for individual species with $\geq$ 3000 observations.

$$p_{live} = 1 - e^{-e^{-\beta_1 + \beta_2 \frac{dbh}{(bal+1)}}}$$

where: dbh = diameter at breast height (cm), bal = basal area per hectare in larger trees (m^2/ha).

The equation was fit minimizing the trees per hectare (`tph`) prediction error at the end of the remeasurement interval.

For other conifers, data for all conifers were fit to the above equation. For other hardwoods, the following equation was used:

$$p_{live} = 1 - e^{-e^{-\beta_1 + \beta_2 \frac{dbh}{(bal+1)} + \beta_3 shade^{\beta_4}}}$$

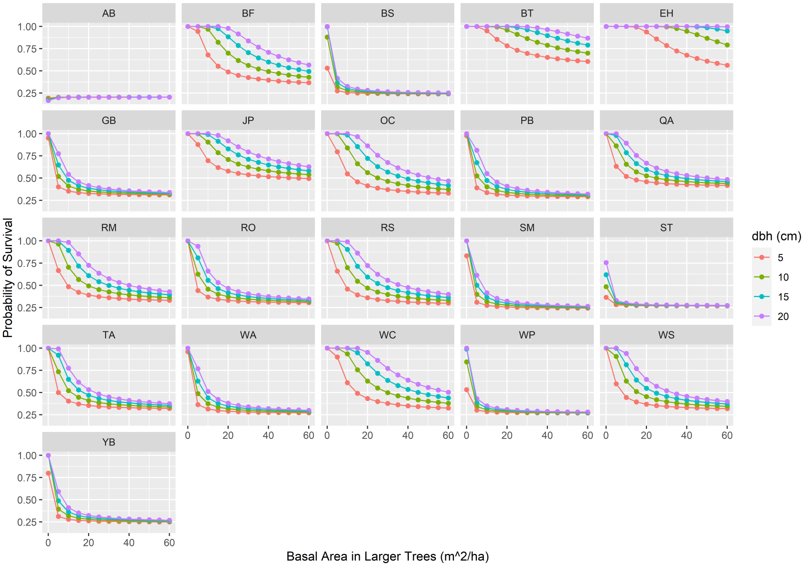
Where `shade` is a shade tolerance metric ranging from 0 (intolerant) to 5 (tolerant) developed by Niinemets and Valladares (2006)[7].

[7] Niinemets, Ülo & Valladares, Fernando. (2006). Tolerance to shade, drought, and waterlogging of temperate Northern Hemisphere trees and shrubs. Ecological Monographs. 76. 521-547. 10.1890/0012-9615(2006)076[0521:TTSDAW]2.0.CO;2.

Survival Equation Parameter Estimates

FVS	n	MSE	b1	b2
BF	192016	4892.91	-0.9848889	2.4538404
RM	80731	3739.91	-1.0227770	1.3414618
RS	77277	1858.39	-1.1598653	1.4769163
BS	53467	895.85	-1.3062797	0.2057468
PB	41097	5023.45	-1.1117057	0.4894070
WS	25287	725.67	-1.0511570	1.1471993
SM	24258	2087.40	-1.3070849	0.3770119
YB	19383	4727.49	-1.2748426	0.3494838
WC	18029	2298.47	-1.1290362	2.3533388
QA	17925	7312.70	-0.6830450	0.8194304
WP	14065	4034.24	-1.1720512	0.1792058
AB	13666	4141.47	-1.4749075	-0.0112790
EH	12502	1521.70	-0.8271663	7.7925149
TA	6468	1080.67	-1.0124098	0.7754996
RO	5106	3369.09	-1.0641367	0.6281519
GB	5105	13011.32	-1.0251715	0.4273021
BT	3657	10090.58	-0.3307043	3.1555813
WA	3506	6984.87	-1.1872563	0.4704159
ST	3299	39200.67	-1.1681473	0.0755869
JP	3203	54.82	-0.5029027	1.4967724
OC	403960	3184.89	-1.0657725	1.8371527

Individual Tree Survival

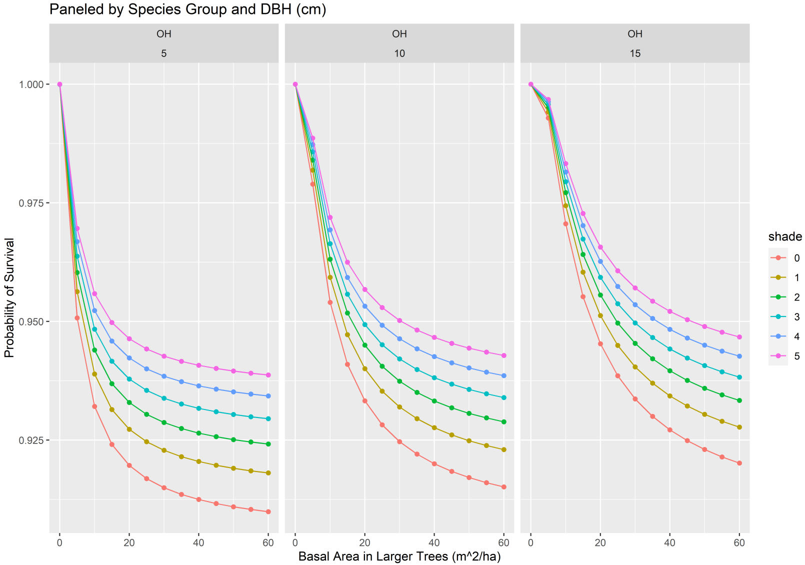


Other Hardwood Survival Equation Parameter Estimates

Survival Equation Parameter Estimates

FVS	n	MSE	b1	b2	b3	b4
OH	227776	6290.5	-0.853984	0.297958	0.1299751	0.8353978

Other Hardwood Individual Tree Survival



Change in Height to Crown Base

`ACD` uses a dynamic crown recession equation developed by Russell et al.[8] The equation form is:

$$\Delta hcb = \frac{cl + \Delta ht}{1 + e^{\beta_1 + \beta_2 log(cr+0.01) + \beta_3 log(ccf+1) + \beta_4 log(1.01-cr) + \beta_5 shade + \beta_6 log(shade \times cr)}}$$

where `cl` is crown length and `ccf` is crown competition factor (m^2/ha).

In verification work using `ACD` it became clear that crown dynamics were driving some of the misbehavior in the diameter and height growth functions, so we decided to revise the model.

[8] Russell, Matthew & Weiskittel, Aaron & Kershaw, John. (2014). Comparing strategies for modeling individual-tree height and height-to-crown base increment in mixed-species Acadian forests of northeastern North America. European Journal of Forest Research. 133. 10.1007/s10342-014-0827-1.

Revised Change in Height to Crown Base Equation

We formulated a Δcr equation using height to crown base (`hcb`) and Δht‘:

$$\Delta hcb = \beta_0((ht - hcb) + (\Delta ht)^{\beta_1})$$

In this equation, β_0 is essentially the percentage of the crown length (after height growth) that will be lost in a year.

Many alternative formulations were tried; all of which had similar fit statistics, but dramatically different behavior.

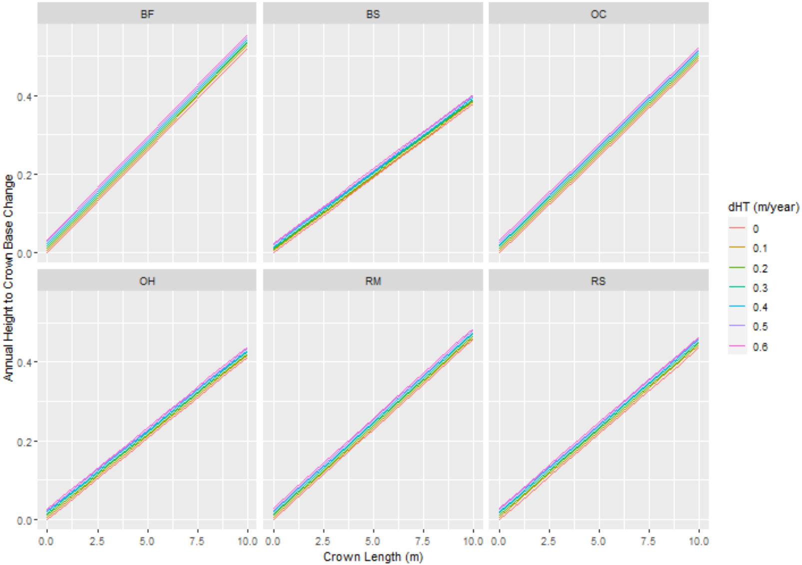
Revised Change in Height to Crown Base Equation

Change in Crown Ratio Equation Parameters

FVS	N	MSE	b0	b1
BF	37047	1.52	0.0521181	0.9098787
RS	15724	1.22	0.0437697	0.8281653
RM	7799	1.45	0.0458042	0.9395264
BS	4166	1.45	0.0379165	0.9792704
OC	63433	1.46	0.0492422	0.9280806
OH	15287	1.45	0.0412473	0.8592384

Revised Change in Height to Crown Base Equation

Change in Height to Crown Base Equation Behavior by Species



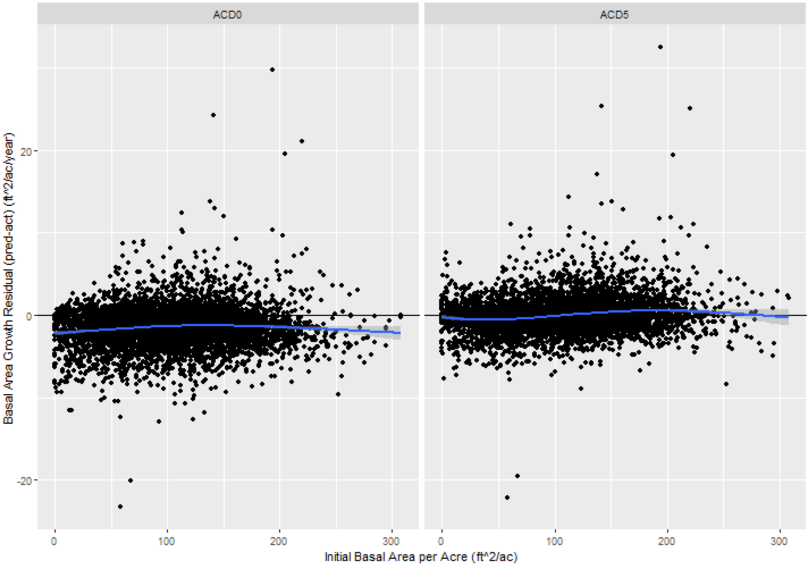
The Bottom Line

After combining all the revised equations, bug fixes, and performance enhancements, was it worth the trip?

To answer that we used a benchmarking data set compiled by Ben Rice. The data set is comprised of 4844 FIA plots with remeasurement data and is mostly independent of the modeling data set. Remeasurement intervals ranged from 5 to 21 years.

What follows are side-by-side comparisons of the C++ version of `ACD` (called `ACD0`) and the revised model (`ACD5`).

Benchmark Validation



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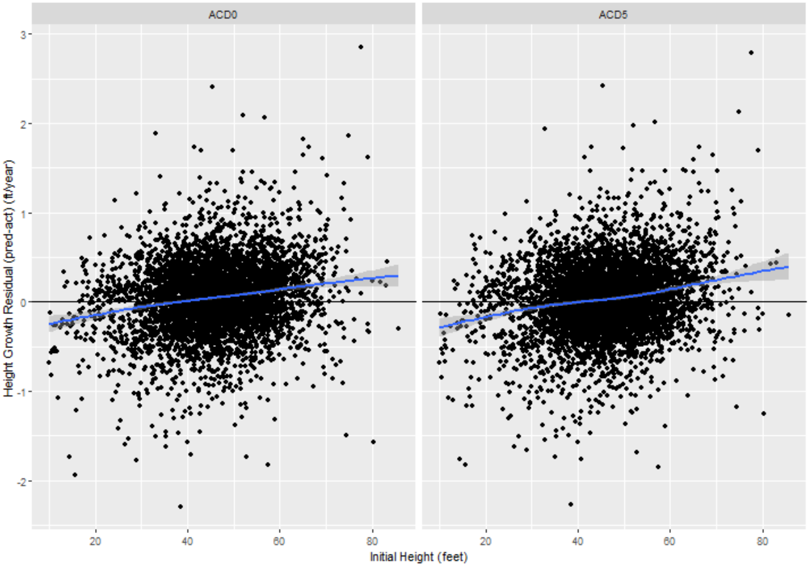
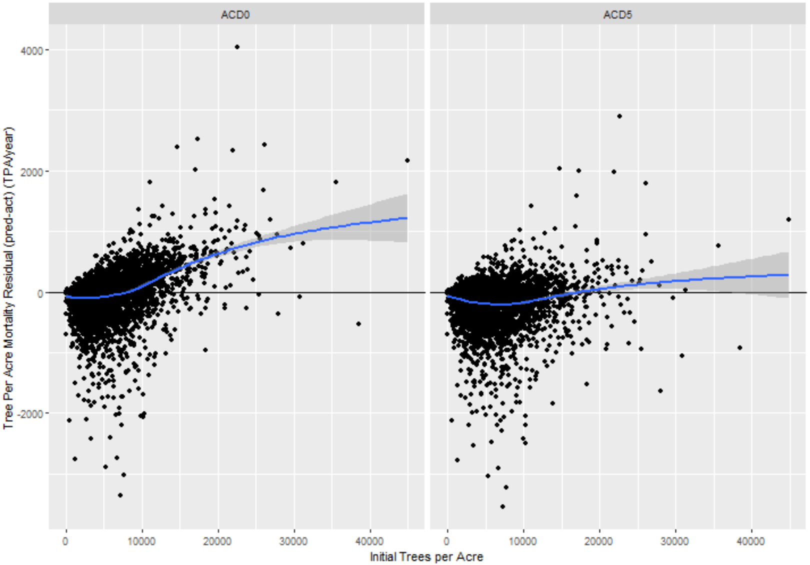
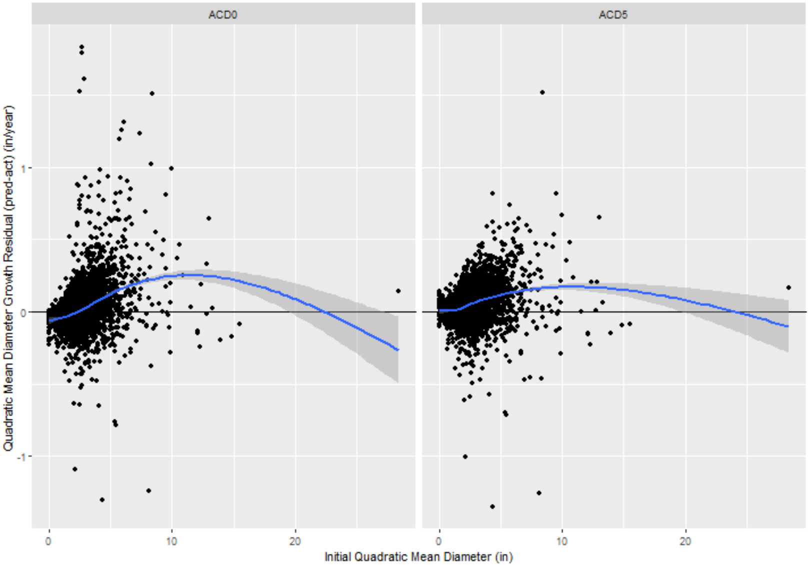


Benchmark Validation



Benchmark Validation



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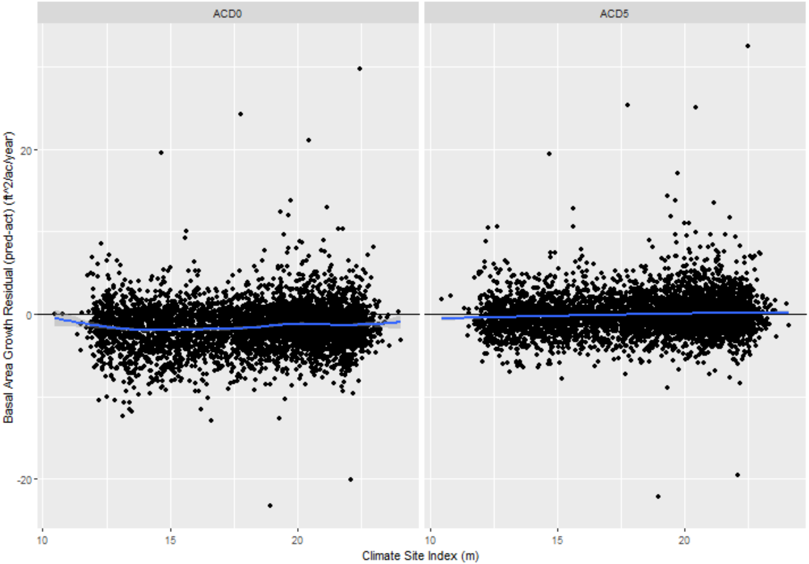

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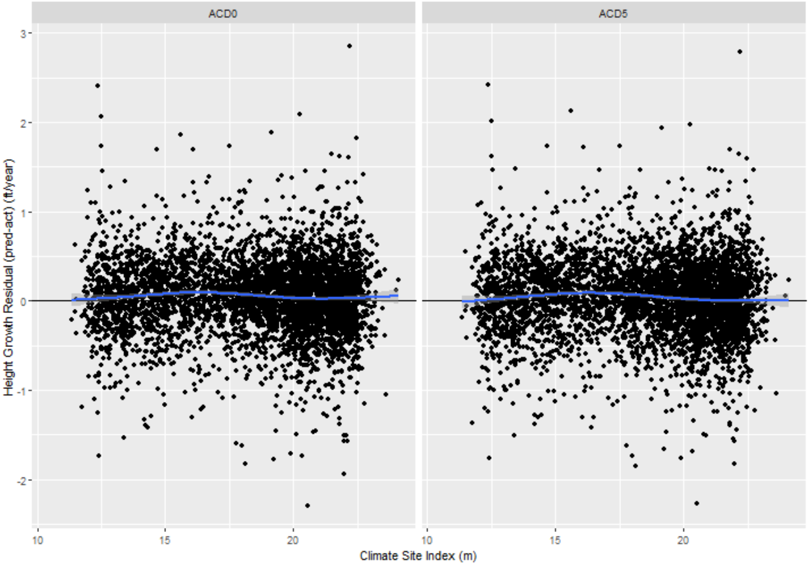

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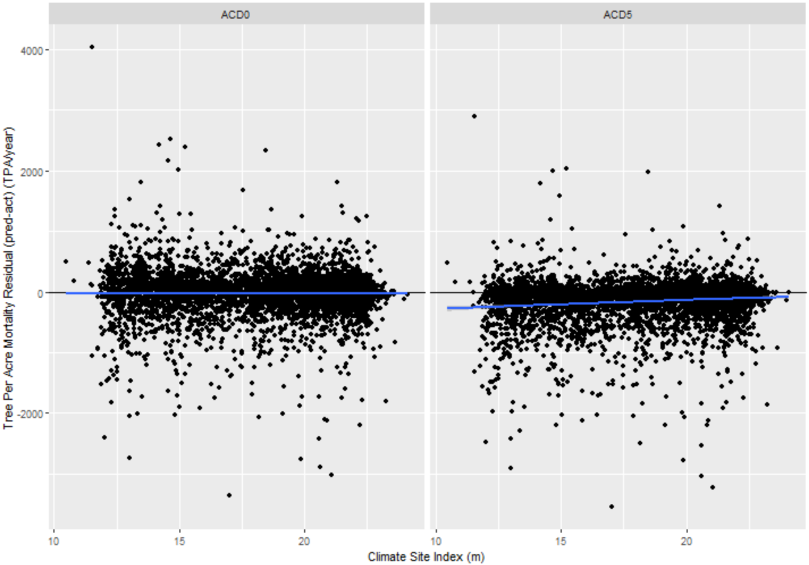

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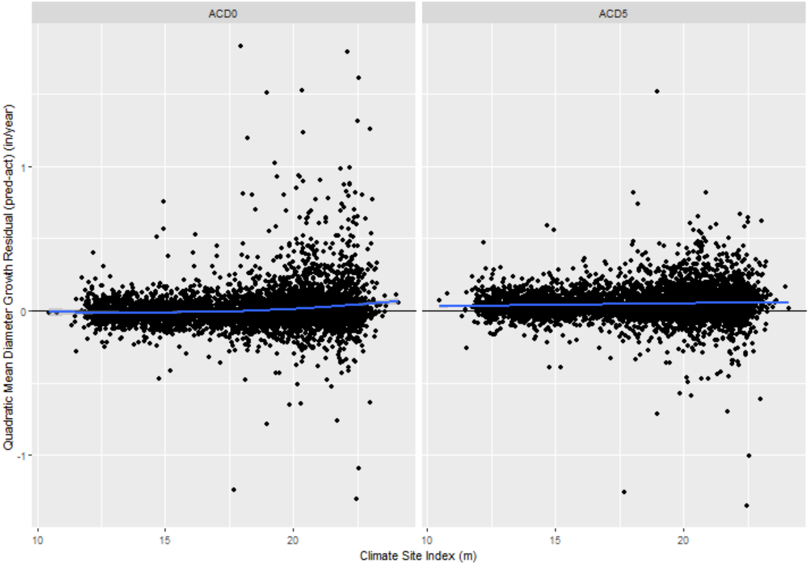


Benchmark Validation

Table 6: Annual Growth Residuals

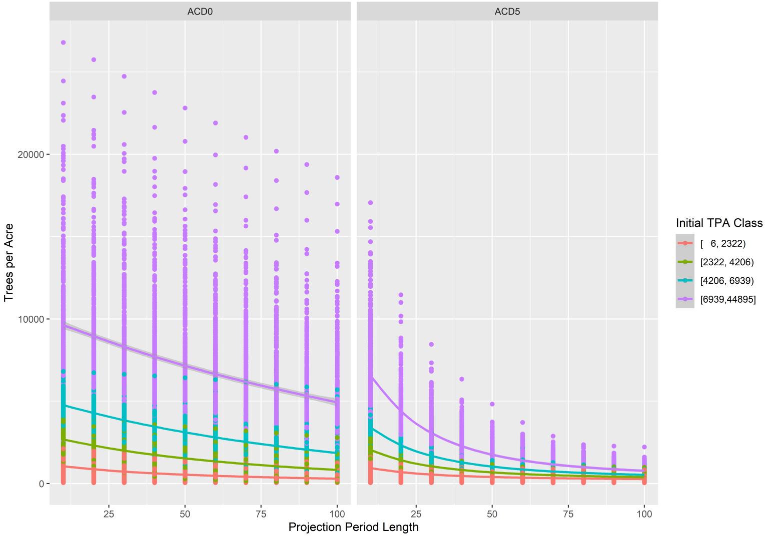
Variable	N	Mean	Median	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
version: ACD0								
Basal Area Growth Error	4844	-1.49	-1.37	2.64	-23.3	-2.86	-0.0559	29.8
Tree per Acre Change Error	4844	-30.6	-5.77	375	-3354	-137	118	4038
Quadratic Mean Diameter Growth Error	4844	0.00844	-0.0112	0.152	-1.31	-0.0609	0.0457	1.83
Height Growth Error	4616	0.0446	0.0537	0.415	-2.3	-0.19	0.284	2.85
version: ACD5								
Basal Area Growth Error	4844	-0.0323	-0.16	2.35	-22.1	-1.28	0.986	32.5
Tree per Acre Change Error	4844	-156	-74.8	350	-3551	-240	7.32	2888
Quadratic Mean Diameter Growth Error	4844	0.0468	0.0365	0.115	-1.35	-0.00797	0.0942	1.52
Height Growth Error	4616	0.0314	0.0455	0.421	-2.28	-0.207	0.278	2.78

Long-term Projection Behavior

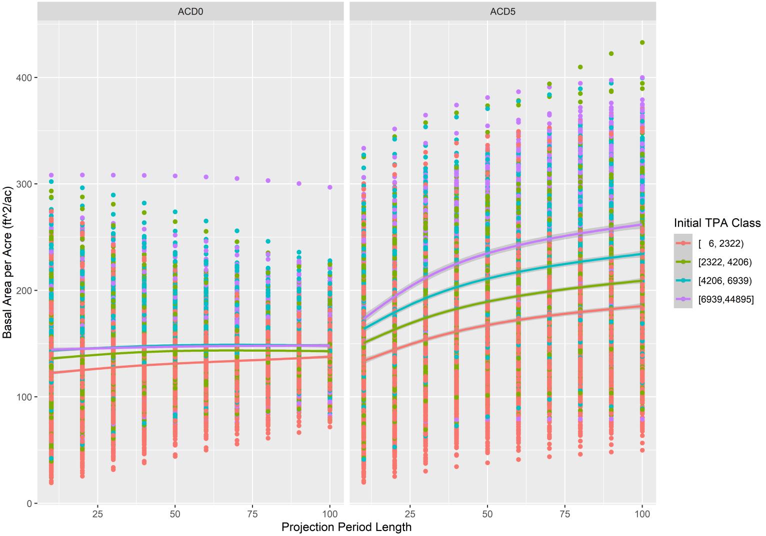
One of the criteria for `ACD` is to insure biologically reasonable behavior over long (100-year) projections. Many of the adjustments in `ACD` (plot-level basal area growth constraint, height growth cap, mortality modifiers) came from the quest to *tame* long-term projections.

The following slides show the performance of `ACD5` for the projection of 2993 plots from the benchmarking data set.

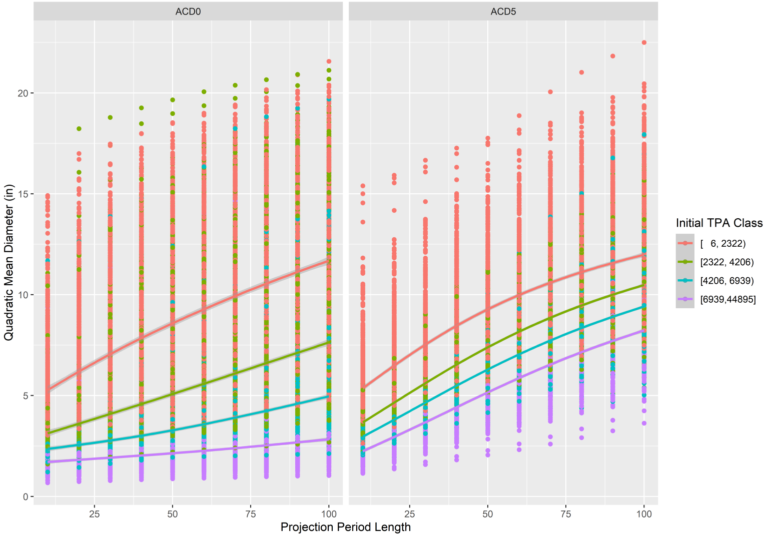
Long-term Projection Behavior



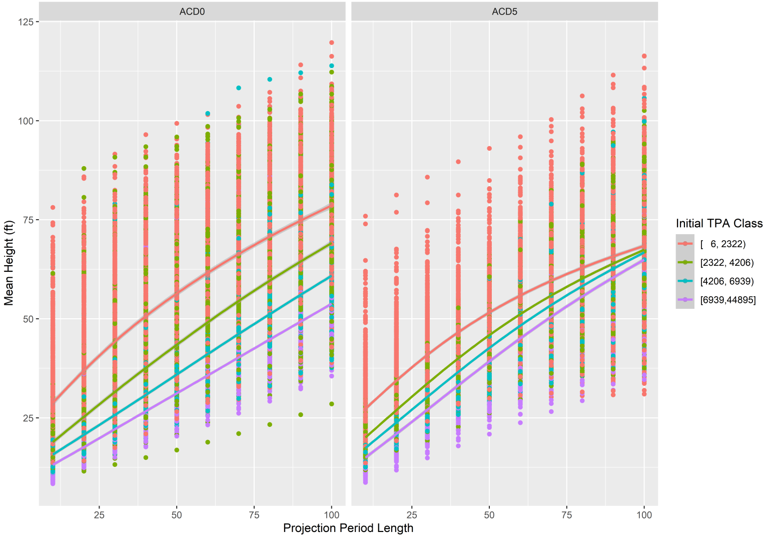
Long-term Projection Behavior



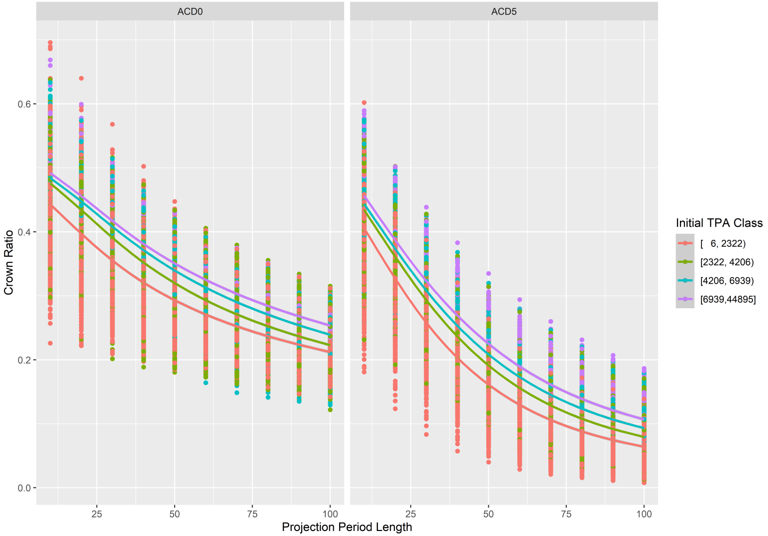
Long-term Projection Behavior



Long-term Projection Behavior



Long-term Projection Behavior



The Bad and the Good

What's Not to Like?

- White Pine diameter and height growth – too fast, too tall
- Apparent under-estimate of individual tree mortality on lower sites and stocking around 5000 trees per hectare
- Apparent over-estimate of stands/plots with small trees (less than 5m tall)
- Lack of algorithm to handle poor sample tree lists

What's to Like?

- C++ code results in high processing speed and opportunities for greater optimization
- R Package will be available using compiled code
- Mostly removed the need for arbitrary limits and other constraints on growth estimates
- Validation on independent data is good (better than the NE Variant of FVS) and can get better

Work To Do

- Ingrowth
- Small tree growth
- High expansion factor tree record management
- Chasing trends in residuals
- Fully diagnosing high basal area attainment issues
- Substitutes or augmentation of climate site index

Thanks!!!